

First, how many ways are there to get two 6's, two 5's, and one 3 if the dice are distinct?

6 6 5 5 3

Ways to arrange five elements

$$\frac{5!}{2! \cdot 2!} = \frac{120}{4} = 30$$

Each 2! accounts for the repeated numbers.
One 2! for the two 6's
One 2! for the two 5's

There are 30 ways to get two 6's, two 5's, and one 3.

Let's declare event W = winning.

$$P(W) = \frac{30}{6^5} = \frac{30}{7776}$$

And we will call event W^c = not winning

$$P(W^c) = 1 - \frac{30}{7776} = \frac{7776}{7776} - \frac{30}{7776} = \frac{7746}{7776}$$

Case 1: Winning on the first try.

$$P(W) = \frac{30}{7776}$$

Case 2: Not winning on attempt 1, but then winning on attempt 2

$$P(W^c, \text{ then } W) = \left(\frac{7746}{7776}\right) \left(\frac{30}{7776}\right)$$

Case 3: Not winning on attempt 1, not winning on attempt 2, but then winning on attempt 3

$$P(W^c, W^c, \text{then } W) = \left(\frac{7746}{7776}\right) \left(\frac{7746}{7776}\right) \left(\frac{30}{7776}\right)$$

Since it is impossible for different cases to be simultaneously true (Cases 1, 2, and 3 are mutually exclusive), we combine the overall chance of getting a win in three attempts by summing the probabilities of each case.

$$\begin{aligned} &P(\text{Case 1 or Case 2 or Case 3}) \\ &= \frac{30}{7776} + \left(\frac{7746}{7776}\right) \left(\frac{30}{7776}\right) + \left(\frac{7746}{7776}\right) \left(\frac{7746}{7776}\right) \left(\frac{30}{7776}\right) \\ &= \frac{30}{7776} \left[1 + \left(\frac{7746}{7776}\right) + \left(\frac{7746}{7776}\right) \left(\frac{7746}{7776}\right)\right] \\ &= \frac{25097165}{2176782336} \approx 0.011529478434 \end{aligned}$$

http://www.wolframalpha.com/input/?i=%2830%2F7776%29%2B%287746%2F7776%29*%2830%2F7776%29%2B%287746%2F7776%29*%287746%2F7776%29*%2830%2F7776%29

The screenshot shows the WolframAlpha interface. At the top is the WolframAlpha logo with the tagline "computational... knowledge engine". Below the logo is a search bar containing the input expression: $(30/7776) + (7746/7776) * (30/7776) + (7746/7776) * (7746/7776) * (30/7776)$. Below the search bar are navigation icons and links for "Examples" and "Random". The main content area displays the input, the exact result as a fraction $\frac{25097165}{2176782336}$, and a decimal approximation $0.011529478434723939251958354737402646766056815337920860452994...$. There are buttons for "Step-by-step solution" and "More digits".