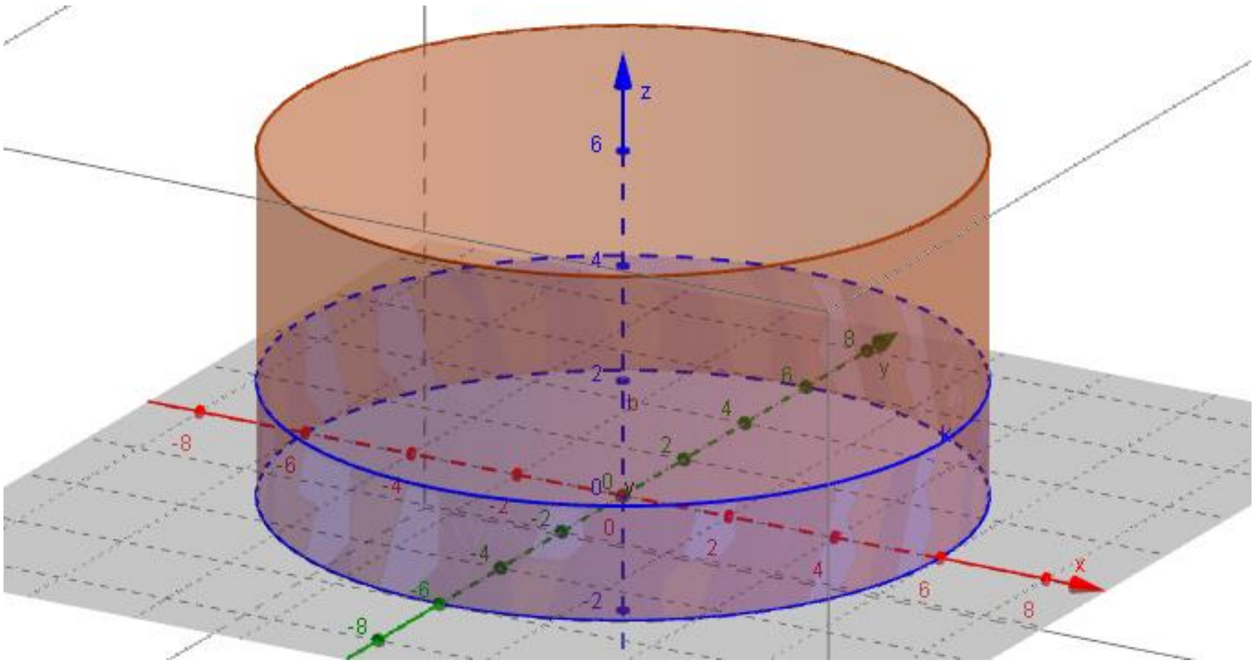
Question 3. A cylindrical can with a radius of 6 inches is filled with 2 inches of water. When a smaller cylindrical can 4 inches tall is placed inside of the first with its bottom lying on the bottom of the first, the water in the first can rises a further inch to 3 inches. What is the radius of the smaller can?

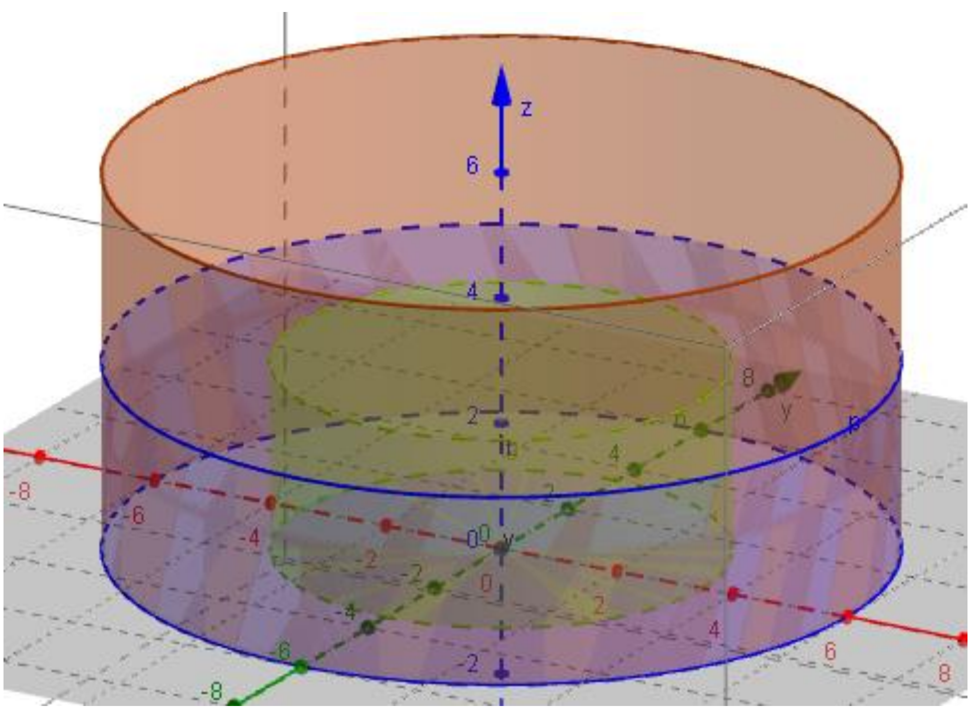
(A) 3 (B) $\sqrt{12}$ (C) $\sqrt{15}$ (D) 4 (E) $\sqrt{18}$

Strategy:

Here's a model of the original water level.



Here's a model of the water level after the small green can is added.



We need to know the volume of a cylinder which is given by $V = \pi r^2 h$ where $r =$ the radius of the cylinder and $h =$ the height of the cylinder.

The volume of the water in the can is $\pi(6^2)(2) = 72\pi$.

The height of the smaller can is 3. This means the volume of the smaller can is $V = \pi(r^2)(3) = 3\pi r^2$.

When we add the smaller can, its volume takes away from the space the water can occupy, driving the water level to rise from 2 inches to 3 inches. The new water level including the small can is that of a cylinder with the same volume as a can of height 3 and radius 6. This volume would be $\pi(6^2)(3) = 108\pi$.

The volume of the water combined with the small can is given by $108\pi - 3\pi r^2$. Notice the amount of water present did not change. We need $108\pi - 3\pi r^2$ to be the same as the volume of the water, 72π.

$108\pi - 3\pi r^2 = 72\pi$

$36\pi - 3\pi r^2 = 0$

$3\pi(12 - r^2) = 0$

$12 - r^2 = 0$

$12 = r^2$

$r = \pm\sqrt{12}$

$r = \sqrt{12}$ (since the radius cannot be negative)